FLOW VISUALIZATION

1. Path lines or Particle paths – paths followed by fluid particles,
2. Streaklines – locus of Lagrangian fluid particles passing through a given point,
3. Time lines – locus of Lagrangian fluid particles that formed a given curve at a reference time,
4. Streamlines – curves everywhere tangential to the fluid velocity field.

An example flow
$u=1, \quad v=t, \quad w=0.$
$\underline{u} = (u, v, w)$.

1. Particle paths or Pathlines:

Paths followed by fluid particles.

- Experimental technique :

  (i) Dye or tag a small volume of fluid. Follow the dye/tag.

  (ii) Points visited by the volume constitutes pathline.

  (iii) Use long exposures (old school) or digitally overlap video frames.

- Theoretical determination: (from a given Eulerian velocity field $\underline{u}(\underline{x},t)$)

$$\frac{\partial \underline{F}}{\partial t} = \underline{u}(\underline{F}, t) \quad \text{with} \quad \underline{F} = \underline{X} \text{ at } t=0.$$

(same process as that of determining current location of Lagrangian particles.)

2. Streaklines : Fluid particles that pass through a given Eulerian point

- Experimental technique : Continuously introduce dye from a point in the fluid. The streak formed by the dye is the streakline.

- Theoretical determination : $\underline{F}(\underline{X}, t)$ = Lagrangian position.

  Current position of all Lagrangian particles that passed through a given point. $(\underline{X}_0)$

$$\underline{F}(\underline{X}, s) = \underline{X}_0 \quad \text{for some} \quad t_i \leq s \leq t_f .$$

3. Timelines : Locus of Lagrangian points that coincided with a given curve at some reference time t_0 in the past.

- Experimental technique :

  Release a puff of dye along a curve in the fluid and follow its motion.

- Theoretical determination :

  Parameterize initial curve as $\underline{X}_0(s)$. Then timeline at time t is given by

$$\underline{F}(\underline{X}_0(s), t) \qquad \text{parameterized by } s.$$

4. Streamlines : curves everywhere parallel to the fluid velocity.

- Experimental technique : No direct technique.
- Theoretical determination : (given Eulerian velocity $\underline{u}(\underline{x}, t)$)

  - Let a streamline be $\underline{x}(s)$, parameterized by s.
  - The tangent to the streamline is $\frac{\partial \underline{x}}{\partial s}$ (at fixed t).
  - The condition of tangency implies

$$\frac{\partial \underline{x}}{\partial s} = \underline{u}(\underline{x}(s), t), \quad \text{for every } s.$$